Factorising Quadratics

Quadratic Equations

Standard form of a quadratic equation: $ax^2 + bx + c = 0$

a, b and c can be any number.

To FACTORISE — put it into two brackets.
To SOLVE — find the values of x that make each bracket equal to 0.

Factorising when a = 1

1. Rearrange to $x^2 + bx + c = 0$.
2. Write two brackets: $(x \quad)(x \quad) = 0$
3. Find two numbers that multiply to give 'c' AND add/subtract to give 'b'.
4. Fill in + or – signs.
5. Check by expanding brackets.
6. Solve the equation.

EXAMPLE

Solve $x^2 - 6x = -8$.

1. $x^2 - 6x + 8 = 0$
2. $(x \quad)(x \quad) = 0$
3. Factor pairs of 8: 1×8 or 2×4. $2 + 4 = 6$, so you need 2 and 4.
4. $(x - 2)(x - 4) = 0$
5. $(x - 2)(x - 4) = x^2 - 4x - 2x + 8$ $= x^2 - 6x + 8$
6. $(x - 2) = 0 \Rightarrow x = 2$; $(x - 4) = 0 \Rightarrow x = 4$

Factorising when a is not 1

1. Rearrange to $ax^2 + bx + c = 0$.
2. Write two brackets where the first terms multiply to give 'a'.
3. Find pairs of numbers that multiply to give 'c'.
4. Test each pair in both brackets to find one that adds/subtracts to give 'bx'.
5. Fill in + or – signs.
6. Check by expanding brackets.
7. Solve the equation.

EXAMPLE

Solve $2x^2 + x - 6 = 0$.

1. This is in the standard format.
2. $(2x \quad)(x \quad) = 0$
3. Factor pairs of 6: 1×6 or 2×3
4. $(2x \quad 1)(x \quad 6) \rightarrow 12x$ and x; $(2x \quad 6)(x \quad 1) \rightarrow 2x$ and $6x$; $(2x \quad 2)(x \quad 3) \rightarrow 6x$ and $2x$; $(2x \quad 3)(x \quad 2) \rightarrow 4x$ and $3x$ ($4x - 3x = x$)
5. $(2x - 3)(x + 2) = 0$
6. $(2x - 3)(x + 2)$ $= 2x^2 + 4x - 3x - 6$ $= 2x^2 + x - 6$
7. $(2x - 3) = 0 \Rightarrow x = \frac{3}{2}$; $(x + 2) = 0 \Rightarrow x = -2$

Solving Quadratics

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Use the quadratic formula when:

- the quadratic won't factorise.
- the question mentions d.p. or s.f.
- you need exact answers or surds.

1. Rearrange equation into the form $ax^2 + bx + c = 0$.
2. Identify a, b and c.
3. Substitute into formula.
4. Evaluate both solutions.

Check your answers by substituting back into the original equation.

EXAMPLE

Find the solutions to $4x^2 + 3x = 5$ to 2 d.p.

1. $4x^2 + 3x - 5 = 0$
2. $a = 4, b = 3, c = -5$
3. $x = \frac{-3 \pm \sqrt{3^2 - 4 \times 4 \times -5}}{2 \times 4} = \frac{-3 \pm \sqrt{89}}{8}$ (The ± sign means you get two solutions.)
4. $x = -1.55$ (2 d.p.) or 0.80 (2 d.p.)

Completing the Square

1. Multiply out initial bracket $(x + \frac{b}{2})^2$.
2. Add/subtract adjusting number to match original equation.
3. Set equal to 0 and solve.

EXAMPLE

Solve $x^2 + 4x - 3 = 0$. (Check this is in the standard format first.)

1. $(x + 2)^2 = x^2 + 4x + 4$
2. $(x + 2)^2 - 7 = x^2 + 4x + 4 - 7$ $= x^2 + 4x - 3$ (Add/subtract to get –3.)
3. $(x + 2)^2 - 7 = 0$; $(x + 2)^2 = 7$; $x + 2 = \pm\sqrt{7}$, so $x = -2 \pm \sqrt{7}$

... when a is not 1

1. Take out a factor of 'a' from the first two terms.
2. Multiply out initial bracket $a(x + \frac{b}{2a})^2$.
3. Add/subtract adjusting number to match original equation.

EXAMPLE

Write $2x^2 - 8x + 3$ in the form $a(x + m)^2 + n$. (Check this is in the standard format first.)

1. $2(x^2 - 4x) + 3$
2. $2(x - 2)^2 = 2x^2 - 8x + 8$ (Add/subtract to make this 3.)
3. $2(x - 2)^2 - 5 = 2x^2 - 8x + 8 - 5$ $= 2x^2 - 8x + 3$

When a is positive, the adjusting number tells you the minimum y-value of the graph. This occurs when the brackets = 0, i.e. when $x = -m$. This also gives you the coordinates of the turning point of the graph.

Algebraic Fractions

Simplifying Algebraic Fractions

Cancel terms on the top and bottom.
Deal with one number or letter at a time.

$$\frac{8x^3y}{2x^2y^3} = \frac{^4\cancel{8}\times \cancel{x^3}^{x} \times \cancel{y}}{\cancel{2}\times \cancel{x^2}\times \cancel{y^3}_{y^2}} = \frac{4x}{y^2}$$

÷2 on top and bottom
÷x^2 on top and bottom
÷y on top and bottom

You might have to factorise first, then cancel a common factor:

$$\frac{x^2-x-2}{x^2+5x+4} = \frac{\cancel{(x+1)}(x-2)}{\cancel{(x+1)}(x+4)} = \frac{x-2}{x+4}$$

Multiplying

Multiply tops and bottoms of the fractions separately.

$$\frac{x}{3x+12}\times\frac{2x+8}{x-1} = \frac{x}{3\cancel{(x+4)}}\times\frac{2\cancel{(x+4)}}{x-1} = \frac{x\times 2}{3\times(x-1)} = \frac{2x}{3(x-1)}$$

Factorise and cancel first, to make multiplying easier.

Dividing

To divide, turn the second fraction upside down, then multiply.

$$\frac{x-5}{x^2-9}\div\frac{5x}{x-3} = \frac{x-5}{x^2-9}\times\frac{x-3}{5x} = \frac{x-5}{\cancel{(x-3)}(x+3)}\times\frac{\cancel{x-3}}{5x} = \frac{x-5}{(x+3)\times 5x} = \frac{x-5}{5x(x+3)}$$

Factorise using D.O.T.S.

Adding and Subtracting

1. Find a common denominator.
2. Multiply the top and bottom of each fraction by whatever gives the common denominator.
3. Add or subtract numerators.

The common denominator is something both denominators divide into.

EXAMPLE

Write $\frac{2}{2x-1}-\frac{3}{x+4}$ as a single fraction in its simplest form.

1. Common denominator: $(2x-1)(x+4)$
2. $\frac{2(x+4)}{(2x-1)(x+4)}-\frac{3(2x-1)}{(x+4)(2x-1)}$
3. $=\frac{2(x+4)-3(2x-1)}{(2x-1)(x+4)} = \frac{2x+8-6x+3}{(2x-1)(x+4)}$

$=\frac{11-4x}{(2x-1)(x+4)}$ — Collect like terms together.

Functions

Evaluating Functions

FUNCTION — takes an input, processes it, outputs a value.

They're usually written like:

$f(x) = (x+2)^2 - 5$

This means "take a value of x, add 2, square it, then subtract 5".

Evaluate functions by just substituting in the value of x.

$f(-4) = (-4+2)^2 - 5$
$= (-2)^2 - 5 = -1$

Functions can also be written like $f: x \to (x+2)^2 - 5$.

Composite Functions

COMPOSITE FUNCTION — two functions combined into a single function.

fg(x) → put g(x) into f(x)
gf(x) → put f(x) into g(x)

Three steps for composite functions:

1. Write composite function with brackets.
2. Replace first function with its expression.
3. Substitute it into second function.

EXAMPLE

$f(x) = 4x - 1$ and $g(x) = \frac{3x}{2}$.

a) Find fg(x).

$f(g(x)) = f\left(\frac{3x}{2}\right) = 4\times\frac{3x}{2} - 1 = 6x - 1$

b) Find gf(x).

$g(f(x)) = g(4x-1) = \frac{3(4x-1)}{2} = \frac{12x-3}{2} = 6x - \frac{3}{2}$

In general, $fg(x) \neq gf(x)$.

Inverse Functions

INVERSE FUNCTION, $f^{-1}(x)$ — a function that reverses f(x).

Three steps for inverse functions:

1. Write the equation x = f(y).
2. Make y the subject.
3. Replace y with $f^{-1}(x)$.

EXAMPLE

Given $f(x) = 7x - 11$, find $f^{-1}(x)$.

1. $x = 7y - 11$ — Replace f(x) with x and x with y.
2. $7y = x + 11$
   $y = \frac{x+11}{7}$
3. $f^{-1}(x) = \frac{x+11}{7}$

Check it reverses the function: $f(2) = 3$, and $f^{-1}(3) = 2$ ✓