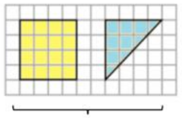
Calculating Space – Part 1

Area of triangles

Area can be calculated by counting squares.
Often this is an estimation with triangles if it does not cut a square in half

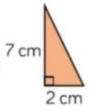


Notice the relationship between the square and the triangle

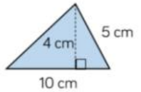
Area triangle = $\frac{1}{2}$ area of the square

Right-angled triangles

The height of a right-angled triangle



Perpendicular heights

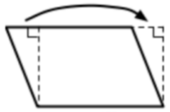


The perpendicular height meets the base at 90°

Area = $\frac{1}{2} \times 10 \times 4$ = 20cm²

Area triangle = $\frac{1}{2}$ x base x perpendicular height

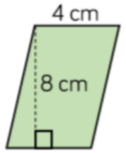
Area of parallelograms



Parallelogram = Base x Perpendicular height

Properties of parallelograms

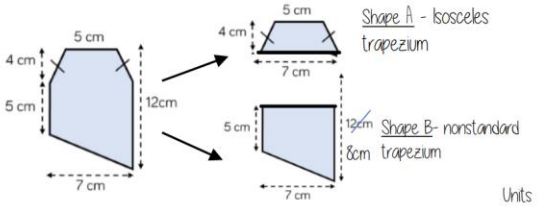
- Two sets of parallel lines
- Four sides (quadrilateral)
- Interior angles = 360°
- Opposite angles are equal
- 2D shape



Area = 4 x 8 = 32cm²

Compound shapes

To find the area compound shapes often need splitting into more manageable shapes first. Identify the shapes and missing sides etc. first.



Units

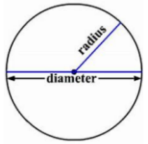
Shape A + Shape B = total area

$\frac{(5+7) \times 4}{2} + \frac{(5+8) \times 7}{2} = 24 + 45.5 = 69.5\text{cm}^2$

Area of a circle (Calculator)

How to get π symbol on the calculator

Area of a circle
π x radius²

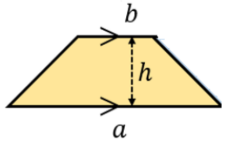


It is important to round your answer suitably – to significant figures or decimal places. This will give you a decimal solution that will go on forever!

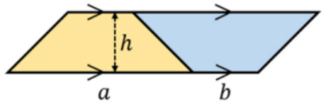
Area of a trapezium

Area of a trapezium

$$\frac{(a+b) \times h}{2}$$



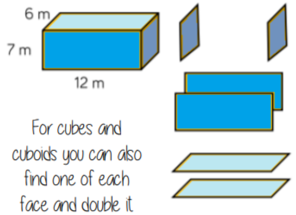
Why?



- Two congruent trapeziums make a parallelogram
- New length (a + b) x height
- Divide by 2 to find area of one

Surface area

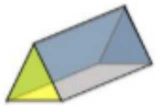
Sketching nets first helps you visualise all the sides that will form the overall surface area



Sides	6 x 7
	6 x 7
Front and back	12 x 7
	12 x 7
Top and Bottom	12 x 6
	12 x 6

Sum of all sides is surface area

For cubes and cuboids you can also find one of each face and double it

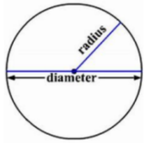


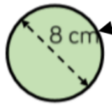
For other shapes = not all the sides are the same, so calculate the individually

Area of a circle (Non-Calculator)

Read the question – leave in terms of π or if $\pi \approx 3$ (provides an estimate for answers)

Area of a circle
π x radius²





Diameter = 8cm
∴ Radius = 4cm

π x radius²
= π x 4²
= π x 16
= 16π cm²

Find the area of one quarter of the circle

Radius = 4cm

Circle Area = 16π cm²
Quarter= 4π cm²

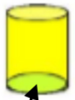
Volumes

Volume is the 3D space it takes up – also known as capacity if using liquids to fill the space

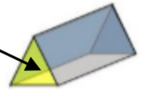
Counting cubes
Some 3D shape volumes can be calculated by counting the number of cubes that fit inside the shape.

Cubes/ Cuboids = base x width x height

Remember multiplication is commutative



Cross section



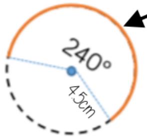
Cross section

Prisms and cylinders
= area cross section x height

Height can also be described as depth

Arc length

Remember an arc is part of the circumference
Circumference of the whole circle = $\pi d = \pi \times 9 = 9\pi$



Arc length = $\frac{\theta}{360}$ ×circumference

$= \frac{240}{360} \times 9\pi$

$= \frac{2}{3} \times 9\pi = 6\pi$

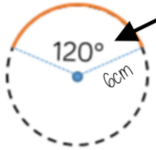
Perimeter

Perimeter is the length around the outside of the shape
This includes the arc length and the radii that encloses the shape

Perimeter = $\frac{\theta}{360}$ ×circumference $+ 2r$ $= 6\pi + 9$

Sector area

Remember a sector is part of a circle
Area of the whole circle = $\pi r^2 = \pi \times 6^2 = 36\pi$



Sector area = $\frac{\theta}{360}$ × area of circle

$= \frac{120}{360} \times 36\pi$

$= \frac{1}{3} \times 36\pi = 12\pi$

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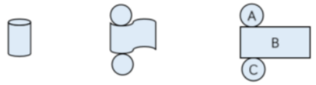
Calculating Space– Part 2

Surface area of cones and cylinders

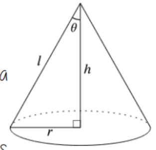
Surface area cylinder= $2\pi r^2 + \pi dh$

Curved surface area Cone = πrl



The area of two circles (top and bottom face) + the area of the curved face

The length of shape B is the circumference of the circles



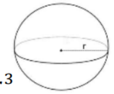
Look out for the use of Pythagoras to calculate the length l

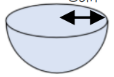
Total surface area = curved face + circle face (area of base)

Volume of a sphere

Volume Sphere = $\frac{4}{3}\pi r^3$

NOTE: This is now a cubed value





3cm

Volume Sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 3^3$

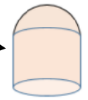
$= \frac{4}{3} \times \pi \times 27 = 36\pi$

A hemisphere is half the volume of the overall sphere

$= 36\pi \div 2$

$= 18\pi$

Look out for hemispheres being placed on other 3D shapes, e.g. cones and cylinders



Calculate the hypotenuse

3 cm (a), 6 cm (b), Hypotenuse

Either of the short sides can be labelled a or b

$a^2 + b^2 = \text{hypotenuse}^2$

1 Substitute in the values for a and b

$3^2 + 6^2 = \text{hypotenuse}^2$

$9 + 36 = \text{hypotenuse}^2$

$45 = \text{hypotenuse}^2$

2 To find the hypotenuse square root the sum of the squares of the shorter sides

$\sqrt{45} = \text{hypotenuse}$

$6.71cm = \text{hypotenuse}$

Calculate missing sides

Hypotenuse 15 cm, b, 12 cm (a)

Either of the short sides can be labelled a or b

$a^2 + b^2 = \text{hypotenuse}^2$

$12^2 + b^2 = 15^2$

1 Substitute in the values you are given

$144 + b^2 = 225$

$-144 \qquad -144$

Rearrange the equation by subtracting the shorter square from the hypotenuse squared

Square root to find the length of the side

$b^2 = 111$

$b = \sqrt{111} = 10.54\ cm$

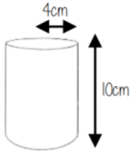
Volume of a cone and a cylinder

Volume Cylinder= $\pi r^2 h$

Volume Cone = $\frac{1}{3}\pi r^2 h$

A cylinder is a prism – cross section is a circle

A cone is a pyramid with a circular base



4cm, 10cm

$V = \pi r^2 h$

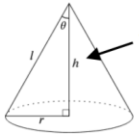
$= \pi \times 4^2 \times 10$

$= \pi \times 160$

$= 160\pi\ cm^3$

Give your answer in terms of 'π' means NOT in terms of pi

$= 502.7cm^3$

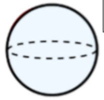


The height of a cone is the perpendicular height from the vertex to the base

Look out for trigonometry or Pythagoras theorem – the radius forms the base of a right-angled triangle

Surface area of a sphere

Surface area = $4\pi r^2$



Radius = 5cm

Surface area = $4\pi r^2$

$= 4 \times \pi \times 5^2$

$= 4 \times \pi \times 25$

The curved surface area of a sphere $= 100\pi$

A hemisphere has the curved surface AND a flat circular face

$= 100\pi \div 2 = 50\pi$

$= 50\pi + \pi \times 5^2$

Hemisphere $= 75\pi$

Hypotenuse, adjacent and opposite

ONLY right-angled triangles are labelled in this way

OPPOSITE – Always opposite an acute angle. Useful to label second. Position depend upon the angle in use for the question

ADJACENT – Next to the angle in question. Often labelled last

HYPOTENUSE – Always the longest side. Always opposite the right angle. Useful to label this first

Tangent ratio: side lengths

$\text{Tan}\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

Substitute the values into the tangent formula

34°, ADJACENT x cm, 10cm OPPOSITE

$\text{Tan}34 = \frac{10}{x}$

Equations might need rearranging to solve

$x \times \text{Tan}34 = 10$

$x = \frac{10}{\text{Tan}34} = 14.8cm$

Sin, Cos, Tan: Angles

Inverse trigonometric functions

ADJACENT 3 cm, OPPOSITE 4 cm, θ

Label your triangle and choose your trigonometric ratio

Substitute values into the ratio formula

$\text{Tan}\theta = \frac{3}{4}$

$\theta = \text{Tan}^{-1}\frac{3}{4}$

$\theta = 36.9°$

$\theta = \text{Tan}^{-1}\frac{\text{opposite side}}{\text{adjacent side}}$

$\theta = \text{Sin}^{-1}\frac{\text{opposite side}}{\text{hypotenuse side}}$

$\theta = \text{Cos}^{-1}\frac{\text{adjacent side}}{\text{hypotenuse side}}$

Key angles 0° and 90°

Tan0 = 0

~~Tan90~~

This value cannot be defined – it is impossible as you cannot have two 90° angles in a triangle

1cm, $\sqrt{2}$ cm, 1cm

Sin0 = 0 | Sin90 = 1

Cos0 = 1 | Cos90 = 0

Key angles

Because trig ratios remain the same for similar shapes you can generalise from the following statements

This side could be calculated using Pythagoras

$\sqrt{3}$ cm, 2 cm, 60°, 1 cm

Tan30 = $\frac{1}{\sqrt{3}}$ | Cos30 = $\frac{\sqrt{3}}{2}$ | Sin30 = $\frac{1}{2}$

Tan60 = $\sqrt{3}$ | Cos60 = $\frac{1}{2}$ | Sin60 = $\frac{\sqrt{3}}{2}$

1 cm, 45°, $\sqrt{2}$ cm, 45°, 1 cm

Tan45 = 1 | Cos45 = $\frac{1}{\sqrt{2}}$ | Sin45 = $\frac{1}{\sqrt{2}}$

Sin and Cos ratio: side lengths

OPPOSITE x cm, 50°, 12 cm HYPOTENUSE

$\text{Sin}\theta = \frac{\text{opposite side}}{\text{hypotenuse side}}$

NOTE: The Sin(x) ratio is the same as the Cos(90-x) ratio

ADJACENT x cm, 40°, 12 cm HYPOTENUSE

$\text{Cos}\theta = \frac{\text{adjacent side}}{\text{hypotenuse side}}$

Substitute the values into the ratio formula

Equations might need rearranging to solve

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