Manipulating Surds

Surds are expressions with irrational square roots in them (remember from p.2 that irrational numbers are ones which can't be written as fractions, such as most square roots, cube roots and π).

Manipulating Surds — 6 Rules to Learn (Grade 7)

There are 6 rules you need to learn for dealing with surds...

1. $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$ e.g. $\sqrt{2} \times \sqrt{3} = \sqrt{2 \times 3} = \sqrt{6}$ — also $(\sqrt{b})^2 = \sqrt{b} \times \sqrt{b} = \sqrt{b \times b} = b$
2. $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ e.g. $\frac{\sqrt{8}}{\sqrt{2}} = \sqrt{\frac{8}{2}} = \sqrt{4} = 2$
3. $\sqrt{a} + \sqrt{b}$ — DO NOTHING — in other words it is definitely NOT $\sqrt{a+b}$
4. $(a + \sqrt{b})^2 = (a + \sqrt{b})(a + \sqrt{b}) = a^2 + 2a\sqrt{b} + b$ — NOT just $a^2 + (\sqrt{b})^2$ (see p.18)
5. $(a + \sqrt{b})(a - \sqrt{b}) = a^2 + a\sqrt{b} - a\sqrt{b} - (\sqrt{b})^2 = a^2 - b$ (see p.19).
6. $\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$ This is known as 'RATIONALISING the denominator' — it's where you get rid of the $\sqrt{\ }$ on the bottom of the fraction. For denominators of the form $a \pm \sqrt{b}$, you always multiply by the denominator but change the sign in front of the root (see example 3 below)

Use the Rules to Simplify Expressions (Grade 8)

EXAMPLES:

1. Write $\sqrt{300} + \sqrt{48} - 2\sqrt{75}$ in the form $a\sqrt{3}$, where a is an integer.

Write each surd in terms of $\sqrt{3}$:

$\sqrt{300} = \sqrt{100 \times 3} = \sqrt{100} \times \sqrt{3} = 10\sqrt{3}$

$\sqrt{48} = \sqrt{16 \times 3} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$

$2\sqrt{75} = 2\sqrt{25 \times 3} = 2 \times \sqrt{25} \times \sqrt{3} = 10\sqrt{3}$

Then do the sum (leaving your answer in terms of $\sqrt{3}$):

$\sqrt{300} + \sqrt{48} - 2\sqrt{75} = 10\sqrt{3} + 4\sqrt{3} - 10\sqrt{3} = 4\sqrt{3}$

2. A rectangle with length $4x$ cm and width x cm has an area of 32 cm². Find the exact value of x, giving your answer in its simplest form.

Area of rectangle = length × width = $4x \times x = 4x^2$

So $4x^2 = 32$

$x^2 = 8$

$x = \pm\sqrt{8}$ — You can ignore the negative square root (see p.22) as length must be positive.

'Exact value' means you have to leave your answer in surd form, so get $\sqrt{8}$ into its simplest form:

$\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$ So $x = 2\sqrt{2}$

3. Write $\frac{3}{2+\sqrt{5}}$ in the form $a + b\sqrt{5}$, where a and b are integers. (Grade 9)

To rationalise the denominator, multiply top and bottom by $2 - \sqrt{5}$:

$\frac{3}{2+\sqrt{5}} = \frac{3(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})}$

$= \frac{6 - 3\sqrt{5}}{2^2 - 2\sqrt{5} + 2\sqrt{5} - (\sqrt{5})^2}$

$= \frac{6-3\sqrt{5}}{4-5} = \frac{6-3\sqrt{5}}{-1} = -6 + 3\sqrt{5}$

(so a = −6 and b = 3)

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