6.4 Factorising — Quadratics

A **quadratic expression** is one of the form $ax^2 + bx + c$ where a, b and c are constants ($a \neq 0$).
You can **factorise** some quadratics.

Example 1

Factorise: a) $x^2 + 6x + 8$ b) $x^2 + 2x - 15$

1. Find two numbers that add up to 6 and multiply to 8:
   a) $4 + 2 = 6$ and $4 \times 2 = 8$
   Then $x^2 + 6x + 8 = \underline{(x + 4)(x + 2)}$
2. You can check this by expanding the brackets.
   $(x + 4)(x + 2) = x^2 + 2x + 4x + 8$

1. You need two numbers that multiply to −15, so one must be positive and the other must be negative.
   b) $5 + -3 = 2$ and $5 \times -3 = -15$
2. They add up to +2, so the positive one is bigger.
   Then $x^2 + 2x - 15 = \underline{(x + 5)(x - 3)}$

Simplifying Algebraic Fractions

Algebraic fractions can be simplified by **factorising** the numerator and/or the denominator, and then cancelling out any **common factors**. For algebraic fractions with **quadratic expressions**, factorise the quadratics **first**, then look for any factors that can be cancelled out.

Example 1

Simplify $\frac{3x^2y^4}{21xy^5}$.

1. 3 is a common factor, so it can be cancelled.
2. x is a common factor, so it can be cancelled.
3. y^4 is a common factor, so it can be cancelled.

$$\frac{3x^2y^4}{21xy^5} = \frac{\cancel{3} \times x^2 \times y^4}{\cancel{3} \times 7 \times x \times y^5} = \frac{x^{\cancel{2}} \times y^4}{7 \times \cancel{x} \times y^5} = \frac{x \times \cancel{y^4}}{7 \times y^{\cancel{5}}} = \frac{x}{7 \times y} = \mathbf{\frac{x}{7y}}$$

Example 2

Simplify $\frac{x^2 - 16}{x^2 + 8x + 16}$.

1. Factorise the numerator and the denominator.
2. Cancel any common factors.

$$\frac{x^2 - 16}{x^2 + 8x + 16} = \frac{\cancel{(x+4)}(x-4)}{\cancel{(x+4)}(x+4)} = \mathbf{\frac{x-4}{x+4}}$$

Adding and Subtracting Algebraic Fractions

Algebraic fractions should be treated in just the same way as numerical fractions. This means that to add or subtract them, they need to have a **common denominator**.

Example 3

Express $\frac{x-2}{3} + \frac{2x+3}{4}$ as a single fraction.

1. The fractions are being added, so first find a common denominator.
2. Here, the common denominator is $4 \times 3 = 12$.
3. Then convert both fractions into fractions with denominator 12, so they can be added.

$$\frac{x-2}{3} + \frac{2x+3}{4} = \frac{4(x-2)}{12} + \frac{3(2x+3)}{12} = \frac{4(x-2) + 3(2x+3)}{12} = \frac{4x - 8 + 6x + 9}{12} = \mathbf{\frac{10x+1}{12}}$$

Multiplying and Dividing Algebraic Fractions

Example 6

Express $\frac{x-2}{x^2+6x+8} \times \frac{2x+4}{x^2+2x-8}$ as a single fraction.

1. Factorise each term as far as possible.
2. Cancel any factor which appears on the top of either fraction and on the bottom of either fraction.
3. Multiply the terms.

$$\frac{x-2}{x^2+6x+8} \times \frac{2x+4}{x^2+2x-8} = \frac{\cancel{x-2}}{\cancel{(x+2)}(x+4)} \times \frac{2\cancel{(x+2)}}{\cancel{(x-2)}(x+4)} = \frac{1}{(x+4)} \times \frac{2}{(x+4)} = \mathbf{\frac{2}{(x+4)^2}}$$

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18.1 Functions

A **function** is a rule that turns one number (the **input**) into another number (the **output**).

For example, the function $f(x) = x + 3$ is a rule that adds 3 to the input value x.
This function could also be written as $f: x \to x + 3$.

Example 1

For the function $f(x) = 30 - 2x^2$, find: a) $f(4)$, b) $f(-2)$

To find f(4), substitute $x = 4$ into $30 - 2x^2$.

a) $f(4) = 30 - 2 \times 4^2$
$= 30 - 2 \times 16$
$= 30 - 32 = \underline{-2}$

Substitute $x = -2$ into $30 - 2x^2$.

b) $f(-2) = 30 - 2 \times (-2)^2$
$= 30 - 2 \times 4$
$= 30 - 8 = \underline{22}$

18.2 Composite Functions

If $f(x)$ and $g(x)$ are two functions, then $gf(x)$ is a **composite function**.

$gf(x)$ means 'put x into function f, then put the answer into function g'.
$gf(x)$ is also sometimes written $g(f(x))$.

Example 1

If $f(x) = 3x - 2$ and $g(x) = x^2 + 1$, then:

a) Calculate gf(3)

Calculate the value of f(3),
then use it as the input in g to find gf(3).

$gf(3) = g(3 \times 3 - 2)$
$= g(7) = 7^2 + 1$
$= \underline{50}$

b) Find fg(x)

Replace x with $g(x)$ in the expression for $f(x)$.

$fg(x) = f(x^2 + 1)$
$= 3(x^2 + 1) - 2$
$= 3x^2 + 3 - 2$
$= \underline{3x^2 + 1}$

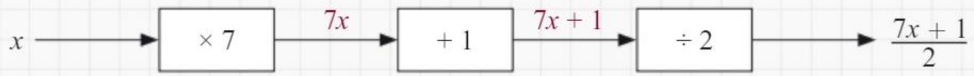
18.3 Inverse Functions

An **inverse function** reverses the effect of a function. 'The inverse of $f(x)$' is written $\mathbf{f^{-1}(x)}$.

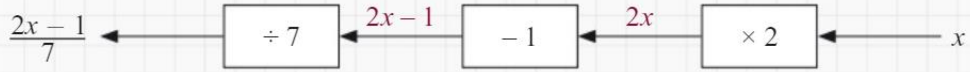
Example 1

Find the inverse of the function $g(x) = \frac{7x + 1}{2}$.

1. Write $g(x)$ out as a function machine.



2. Reverse each step in turn to get the inverse.



So $g^{-1}(x) = \dfrac{2x - 1}{7}$