Simultaneous Equations

What do I need to be able to do?

By the end of this unit you should be able to:

- Determine whether (x,y) is a solution
- Solve by substituting a known variable
- Solve by substituting an expression
- Solve graphically
- Solve by subtracting/ adding equations
- Solve by adjusting equations
- Form and solve linear simultaneous equations

Keywords

Solution: a value we can put in place of a variable that makes the equation true
Variable: a symbol for a number we don't know yet.
Equation: an equation says that two things are equal – it will have an equals sign =
Substitute: replace a variable with a numerical value.
LCM: lowest common multiple (the first time the times table of two or more numbers match)
Eliminate: to remove
Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)
Coordinate: a set of values that show an exact position.
Intersection: the point two lines cross or meet.

Is (x, y) a solution?

x and y represent values that can be substituted into an equation

Does the coordinate (1,8) lie on the line y=3x+5?

This coordinate represents x=1 and y=8

$y = 3x + 5$

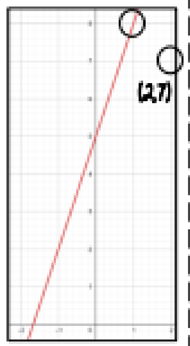
$8 = 3(1) + 5$

As the substitution makes the equation correct the coordinate (1,8) IS on the line y=3x+5

Is (2,7) on the same line?

$7 \neq 3(2) + 5$

No 7 does NOT equal 6+5



Substituting known variables

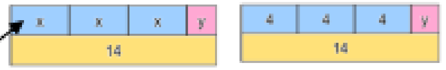
A line has the equation 3x + y = 14

Stephanie knows the point x = 4 lies on that line. Find the value for y.

$x = 4$

$3x + y = 14$

$3(4) + y = 14$



Two different variables, two solutions

$12 + y = 14$
$-12 \quad -12$
$y = 2$

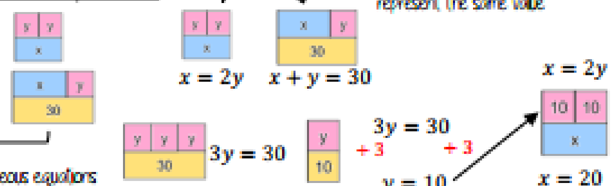
Substituting in an expression

x = 2y

x + y = 30

Pair of simultaneous equations (two representations)

Substitute 2y in place of the x variable as they represent the same value



$x = 2y \qquad x + y = 30$

$3y = 30$

$3y = 30$
$\div 3 \quad \div 3$
$y = 10$

$x = 2y$

$x = 20$

Solve graphically

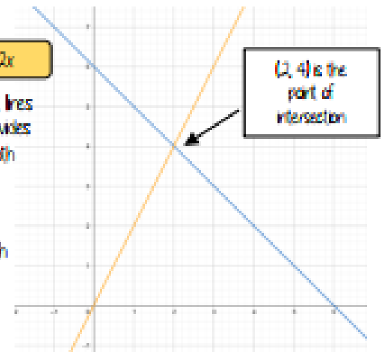
x + y = 6

y = 2x

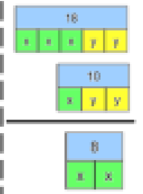
Linear equations are straight lines. The point of intersection provides the x and y solution for both equations

The solution that satisfies both equations is

x = 2 and y = 4



Solve by subtraction

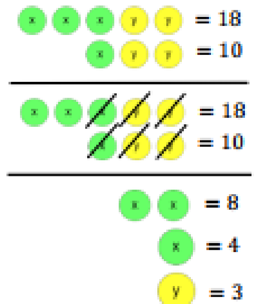


$3x + 2y = 18$
$- \quad x + 2y = 10$

$2x = 8$
$\div 2 \quad \div 2$
$x = 4$

$x + 2y = 10$
$(4) + 2y = 10$
$-4 \quad -4$
$2y = 6$
$\div 2 \quad \div 2$
$y = 3$

$x = 4$
$y = 3$



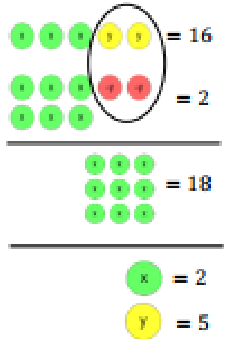
Solve by addition

Addition makes zero pairs

$3x + 2y = 16$
$+ \quad 6x - 2y = 2$

$9x = 18$
$\div 9 \quad \div 9$
$x = 2$

$3x + 2y = 16$
$3(2) + 2(y) = 16$
$6 + 2y = 16$
$-6 \quad -6$
$2y = 10$
$y = 5$

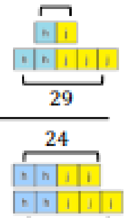


Solve by adjusting one

$h + j = 12$
$2h + 2j = 29$

No equivalent values

$2h + 2j = 24$
$2h + 2j = 29$



By proportionally adjusting one of the equations – now solve the simultaneous equations choosing an addition or subtraction method

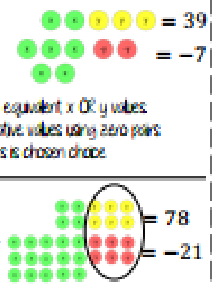
Solve by adjusting both

$2x + 3y = 39$
$5x - 2y = -7$

Use LCM to make equivalent x OR y values. Because of the negative values using zero pairs and y values is chosen choice

$4x + 6y = 78$
$15x - 6y = -21$

Now solve by addition



Addition makes zero pairs

Iterative Methods

Using Iterative Methods

ITERATIVE METHODS — repeating a calculation to get closer to the actual solution. They're used when equations are too hard to solve.

You usually keep putting the value you've just found back into the calculation.

For an equation that equals 0:

Substitute two numbers into the equation. → If the sign changes, there's a solution between the two numbers.

Decimal Search Method

1. Substitute 1 d.p. values of x within the interval until the sign changes.
2. Substitute values of x with 2 d.p. until the sign changes again.
3. Repeat until values either side of the sign change are the same when rounded to required degree of accuracy.

EXAMPLE

The equation $x^3 - 5x - 1 = 0$ has a solution between $x = 0$ and $x = -1$. Find this solution to 1 d.p.

	x	$x^3 - 5x - 1$	Sign
	0	−1	−ve
	−0.1	−0.501	−ve
1	−0.2	−0.008	−ve
	−0.3	0.473	+ve
2	−0.20	−0.008	−ve
	−0.21	0.040739	+ve

3 Both −0.20 and −0.21 round to −0.2 to 1 d.p., so the solution is $x = -0.2$.

Iteration Machines

EXAMPLE

Use the iteration machine below to find a solution to $x^3 - 7x - 2 = 0$ to 1 d.p. Use the starting value $x_0 = 2$.

1 Start with x_n → 2 Find x_{n+1} using the formula $x_{n+1} = \sqrt[3]{7x_n + 2}$ → 3 If $x_{n+1} = x_n$ rounded to 1 d.p., stop. If $x_{n+1} \neq x_n$ rounded to 1 d.p., return to step 1 and repeat with x_{n+1}

Follow the instructions in the iteration machine:

$x_0 = 2$
$x_1 = 2.519... \neq x_0$ to 1 d.p.
$x_2 = 2.697... \neq x_1$ to 1 d.p.
$x_3 = 2.753... \neq x_2$ to 1 d.p.
$x_4 = 2.771... = x_3$ to 1 d.p.

x_n is the nth value, so x_{n+1} is the next value.

x_3 and x_4 both round to 2.8, so the solution is $x = 2.8$ (1 d.p.).

Factorising Quadratics

Quadratic Equations

Standard form of a quadratic equation: $ax^2 + bx + c = 0$

a, b and c can be any number.

To FACTORISE — put it into two brackets.

To SOLVE — find the values of x that make each bracket equal to 0.

Factorising when a = 1

1. Rearrange to $x^2 + bx + c = 0$.
2. Write two brackets: $(x \quad)(x \quad) = 0$
3. Find two numbers that multiply to give 'c' AND add/subtract to give 'b'.
4. Fill in + or – signs.
5. Check by expanding brackets.
6. Solve the equation.

EXAMPLE

Solve $x^2 - 6x = -8$.

1. $x^2 - 6x + 8 = 0$
2. $(x \quad)(x \quad) = 0$
3. Factor pairs of 8: 1×8 or 2×4. $2 + 4 = 6$, so you need 2 and 4.
4. $(x - 2)(x - 4) = 0$
5. $(x - 2)(x - 4) = x^2 - 4x - 2x + 8 = x^2 - 6x + 8$
6. $(x - 2) = 0 \Rightarrow x = 2$
   $(x - 4) = 0 \Rightarrow x = 4$

Factorising when a is not 1

1. Rearrange to $ax^2 + bx + c = 0$.
2. Write two brackets where the first terms multiply to give 'a'.
3. Find pairs of numbers that multiply to give 'c'.
4. Test each pair in both brackets to find one that adds/subtracts to give 'bx'.
5. Fill in + or – signs.
6. Check by expanding brackets.
7. Solve the equation.

EXAMPLE

Solve $2x^2 + x - 6 = 0$. (1) This is in the standard format.

2. $(2x \quad)(x \quad) = 0$
3. Factor pairs of 6: 1×6 or 2×3
4. $(2x \quad 1)(x \quad 6) \rightarrow 12x$ and x
   $(2x \quad 6)(x \quad 1) \rightarrow 2x$ and $6x$
   $(2x \quad 2)(x \quad 3) \rightarrow 6x$ and $2x$
   $(2x \quad 3)(x \quad 2) \rightarrow 4x$ and $3x$ — $4x - 3x = x$
5. $(2x - 3)(x + 2) = 0$
6. $(2x - 3)(x + 2) = 2x^2 + 4x - 3x - 6 = 2x^2 + x - 6$
7. $(2x - 3) = 0 \Rightarrow x = \frac{3}{2}$
   $(x + 2) = 0 \Rightarrow x = -2$

Solving Quadratics

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Use the quadratic formula when:

- the quadratic won't factorise.
- the question mentions d.p. or s.f.
- you need exact answers or surds.

1. Rearrange equation into the form $ax^2 + bx + c = 0$.
2. Identify a, b and c.
3. Substitute into formula.
4. Evaluate both solutions.

Check your answers by substituting back into the original equation.

EXAMPLE

Find the solutions to $4x^2 + 3x = 5$ to 2 d.p.

1. $4x^2 + 3x - 5 = 0$
2. $a = 4, b = 3, c = -5$ — The ± sign means you get two solutions.
3. $x = \frac{-3 \pm \sqrt{3^2 - 4 \times 4 \times -5}}{2 \times 4} = \frac{-3 \pm \sqrt{89}}{8}$
4. $x = -1.55$ (2 d.p.) or 0.80 (2 d.p.)

Completing the Square

1. Multiply out initial bracket $(x + \frac{b}{2})^2$.
2. Add/subtract adjusting number to match original equation.
3. Set equal to 0 and solve.

EXAMPLE

Solve $x^2 + 4x - 3 = 0$. — Check this is in the standard format first.

1. $(x + 2)^2 = x^2 + 4x + 4$
2. $(x + 2)^2 - 7 = x^2 + 4x + 4 - 7 = x^2 + 4x - 3$ — Add/subtract to get –3
3. $(x + 2)^2 - 7 = 0$
   $(x + 2)^2 = 7$
   $x + 2 = \pm\sqrt{7}$, so $x = -2 \pm \sqrt{7}$

... when a is not 1

1. Take out a factor of 'a' from the first two terms.
2. Multiply out initial bracket $a(x + \frac{b}{2a})^2$.
3. Add/subtract adjusting number to match original equation.

EXAMPLE

Write $2x^2 - 8x + 3$ in the form $a(x + m)^2 + n$. — Check this is in the standard format first.

1. $2(x^2 - 4x) + 3$ — Add/subtract to make this 3.
2. $2(x - 2)^2 = 2x^2 - 8x + 8$
3. $2(x - 2)^2 - 5 = 2x^2 - 8x + 8 - 5 = 2x^2 - 8x + 3$

When a is positive, the adjusting number tells you the minimum y-value of the graph. This occurs when the brackets = 0, i.e. when $x = -m$. This also gives you the coordinates of the turning point of the graph.