Type equation h

Addition/ Subtraction laws for indices

$3^5 \times 3^2 = (3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3) \rightarrow 3^7$

The base number is all the same so the terms can be simplified

Addition law for indices

$a^m \times a^n = a^{m+n}$

$3^5 \div 3^2 \rightarrow 3^3$

$\frac{3 \times 3 \times 3 \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3}} \rightarrow \frac{3^3}{3^0} \rightarrow \frac{3^3}{1}$

Subtraction law for indices

$a^m \div a^n = a^{m-n}$

Powers of powers

$(x^a)^b = x^{ab}$

$(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3$

The same base and power is repeated Use the addition law for indices

$(2^3)^4 = 2^{12}$ ← $a \times b = 3 \times 4 = 12$

NOTICE the difference

$(2x^3)^4 = 2x^3 \times 2x^3 \times 2x^3 \times 2x^3$

The addition law applies ONLY to the powers. The integers still need to be multiplied

$(2x^3)^4 = 16x^{12}$

Zero and negative indices

$x^0 = 1$

Any number divided by itself = 1

$\frac{a^6}{a^6} = a^6 \div a^6$

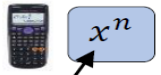
$= a^{6-6} = a^0 = 1$

Negative indices do not indicate negative solutions

$2^2 = 4$
$2^1 = 2$
$2^0 = 1$
$2^{-1} = \frac{1}{2}$
$2^{-2} = \frac{1}{4}$

Looking at the sequence can help to understand negative powers

Higher powers and roots



x^n ← n – power (number of times multiplied by itself)

x – the base number.

$\sqrt[n]{x}$ ← Finding the nth root of any value

Other mental strategies for square roots

$\sqrt{810000} = \sqrt{81} \times \sqrt{10000}$
$= 9 \times 100$
$= 900$

Simplifying Surds

Laws of surds:

$2\sqrt{5} + 3\sqrt{5} = 5\sqrt{5}$

$\sqrt{2} \times \sqrt{3} = \sqrt{6}$

$\sqrt{5} \div \sqrt{2} = \sqrt{\frac{5}{2}}$

$\sqrt{2} \times \sqrt{2} = 2$

Simplify $\sqrt{40}$

Factors of 40::

1	40
2	20
4	10
5	8

In this example we will use 4 and 10 since 4 is the biggest factor which is also a square number:

$\sqrt{4}\sqrt{10} = 2\sqrt{10}$

Remember to always choose the factor pair with the largest square number.

Expanding Brackets with Surds

Single Brackets

$\sqrt{7}(3 + \sqrt{2})$

Multiply both terms in the bracket by $\sqrt{7}$

$3\sqrt{7} + \sqrt{14}$

Double Brackets – Use Foil method or a grid

$(5 - \sqrt{3})(1 + \sqrt{3})$

$5 + 5\sqrt{3} - \sqrt{3} - 3$

$2 + 4\sqrt{3}$

Rationalising the Denominator

$\frac{3}{\sqrt{5}}$

Multiply both the numerator and denominator by $\sqrt{5}$

$\frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$

If the denominator includes a rational number and a surd, swap the + or – sign and then multiply the numerator and denominator by this expression. For example if the denominator includes the expression $3 - \sqrt{2}$, then multiply the numerator and denominator by $3 + \sqrt{2}$.

$\frac{5}{3 - \sqrt{2}}$

$\frac{5}{3 - \sqrt{2}} \times \frac{3 + \sqrt{2}}{3 + \sqrt{2}}$

$\frac{15 + 5\sqrt{2}}{9 + 3\sqrt{2} - 3\sqrt{2} - 2}$

$\frac{15 + 5\sqrt{2}}{7}$