Algebraic constructs

Expression
A sentence with a minimum of two numbers and one maths operation

Equation
A statement that two things are equal

Term
A single number or variable

Identity
An equation where both sides have variables that cause the same answer includes ≡

Formula
A rule written with all mathematical symbols
e.g. area of a rectangle $A = b \times h$

Form expressions

For unknown variables, a letter is normally used in its place

More than – ADD
Less than/ difference – SUBTRACT

e.g. 4 more than t → $t + 4$
8 less than k → $k - 8$

Using letters to represent numbers

$5 + 5 + 5$	$y + y + y + y$	$20 \div h$
3×5	$y \times 4$	$\frac{20}{h}$
5×3	$4 \times y$	
	$4y$	
Addition and multiplication can be done in any order	4 lots of 'y'	20 shared into 'h' number of groups

Commutative calculations

Substitution into expressions

$4y$ ← 4 lots of 'y'

If y = 7 this means the expression is asking for 4 'lots of' 7

4×7 OR $7 + 7 + 7 + 7$ OR 7×4 = 28

e.g. $y - 2$
$= 7 - 2 = 5$

Simplifying expressions

Multiplying algebra – multiply the terms together

$3 \times c = 3c$

$c \times c = c^2$

$6s \times 3t = 18st$

Collecting like terms – only terms that are similar can be collected together. Remember the sign in front of the term goes with that term

$4b + 5b = 9b$

$6c + 3 - 2c + 4 = 4c + 7$

$4a + 2b - 4c + 3a - 4b - 6c = 7a - 2b - 10c$

Expanding single brackets

Everything outside the bracket must be multiplied by everything inside the bracket, be careful with + and – signs when multiplying

$5(c + 2) = 5c + 10$

$-3(v - 4) = -3v + 12$

$x(x - 5) = x^2 - 5x$

Factorising into a single bracket

When we factorise we find the highest common factor or factors in the terms. The highest common factor is put outside the bracket.

$4b + 6 = 2(2b + 3)$

$2 \times 2 \times b + 2 \times 3$ HCF is 2, 2 goes outside the bracket

$a^2 + 6a = a(a + 6)$

$a \times a + a \times 6$ HCF is a, a goes outside the bracket

$24c^2 - 36c = 12c(2c - 3)$

$12 \times 2 \times c \times c - 12 \times 3 \times c$ HCF is 12c, 12c goes outside the bracket

Expand double brackets

Everything in one bracket needs to be multiplied by everything in the other bracket, don't forget to combine like terms.

$(c + 3)(c + 5)$

	c	$+3$
c	c^2	$+3c$
$+5$	$+5c$	$+15$

$(c + 3)(c + 5) = c^2 + 8c + 15$

$(2e - 4)(5e - 3)$

	$2e$	-4
$5e$	$10e^2$	$-20e$
-3	$-6e$	$+12$

$(2e - 4)(5e - 3) = 10e^2 - 26e + 12$

Addition/ Subtraction laws for indices

$3^5 \times 3^2 = (3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3) \rightarrow 3^7$

The base number is all the same so the terms can be simplified

Addition law for indices

$a^m \times a^n = a^{m+n}$

$3^5 \div 3^2 \rightarrow 3^3$

$\frac{3 \times 3 \times 3 \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3}} \rightarrow \frac{3^3}{3^0} \rightarrow \frac{3^3}{1}$

Subtraction law for indices

$a^m \div a^n = a^{m-n}$

Other laws of indices

Anything to the power of 1 is itself

$3^1 = 3$ $400^1 = 400$

$35^1 = 35$ $67^1 = 67$

Anything to the power of 0 is 1

$3^0 = 1$ $400^0 = 1$

$35^0 = 1$ $67^0 = 1$

Multiplication law of indices

$(3^2)^3 \rightarrow 3^6$

$= 3^2 \times 3^2 \times 3^2$

$= 3 \times 3 \times 3 \times 3 \times 3 \times 3$

Multiplication law for indices

$(a^m)^n = a^{m \times n}$