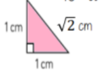
Maths at Southmoor Academy

5N – Investigating properties of shape – Stage 9

Key angles 0° and 90°

Tan0 = 0 | ~~Tan90~~

This value cannot be defined – it is impossible as you cannot have two 90° angles in a triangle

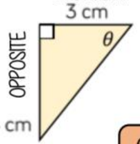


Sin0 = 0 | Sin90 = 1

Cos0 = 1 | Cos90 = 0

Sin, Cos, Tan: Angles

Inverse trigonometric functions



Label your triangle and choose your trigonometric ratio

Substitute values into the ratio formula

$Tan\theta = \frac{3}{4}$

$\theta = Tan^{-1}\frac{3}{4}$

$\theta = 36.9°$

$\theta = Tan^{-1}\frac{\text{opposite side}}{\text{adjacent side}}$

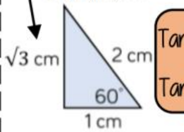
$\theta = Sin^{-1}\frac{\text{opposite side}}{\text{hypotenuse side}}$

$\theta = Cos^{-1}\frac{\text{adjacent side}}{\text{hypotenuse side}}$

Key angles

This side could be calculated using Pythagoras

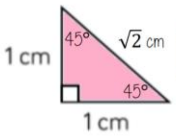
Because trig ratios remain the same for similar shapes you can generalise from the following statements



$Tan30 = \frac{1}{\sqrt{3}}$ | $Tan60 = \sqrt{3}$

$Cos30 = \frac{\sqrt{3}}{2}$ | $Cos60 = \frac{1}{2}$

$Sin30 = \frac{1}{2}$ | $Sin60 = \frac{\sqrt{3}}{2}$



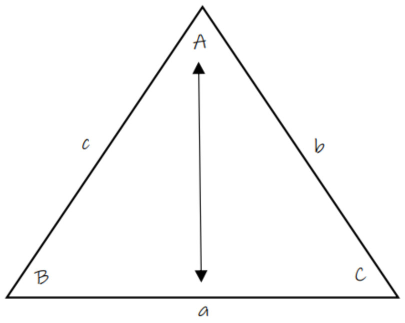
$Tan45 = 1$ | $Cos45 = \frac{1}{\sqrt{2}}$ | $Sin45 = \frac{1}{\sqrt{2}}$

Labelling a non-rightangled triangle

Capital letters are used for the 3 angles

Lower case letters for the 3 sides

Letters of the same type are opposite each other



Sine rule

Used when you know 3 out of these 4 things and need to find the 4th: 2 angles and the 2 sides opposite those angles.

Formula: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ or $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Best used when wanting to find a missing side | Best used when wanting to find a missing angle

Cosine rule

Used when you know 3 out of these 4 things and want to find the 4th: 3 sides and one angle.

Formula: $a^2 = b^2 + c^2 - 2bc\cos A$ or $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

Used to find a missing side. Can be altered to allow you to find "b" or "c" instead.

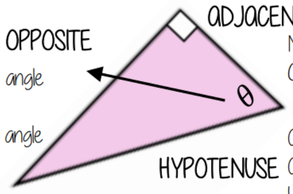
Note: "2bc cosA" is 2 × b × c × cosA

Used to find a missing angle. Can be altered to allow you to find "B" or "C" instead.

Note: Don't forget to do $\cos^{-1}$ to find your actual angle at the end.

Hypotenuse, adjacent and opposite

ONLY right-angled triangles are labelled in this way



OPPOSITE: Always opposite an acute angle. Useful to label second. Position depend upon the angle in use for the question

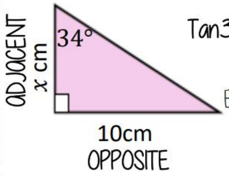
ADJACENT: Next to the angle in question. Often labelled last

HYPOTENUSE: Always the longest side. Always opposite the right angle. Useful to label this first

Tangent ratio: side lengths

$Tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

Substitute the values into the tangent formula



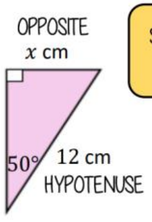
$Tan34 = \frac{10}{x}$

Equations might need rearranging to solve

$x \times Tan34 = 10$

$x = \frac{10}{Tan34} = 14.8cm$

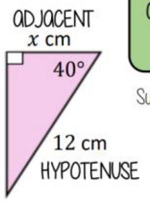
Sin and Cos ratio: side lengths



$Sin\theta = \frac{\text{opposite side}}{\text{hypotenuse side}}$

NOTE

The Sin(x) ratio is the same as the Cos(90-x) ratio



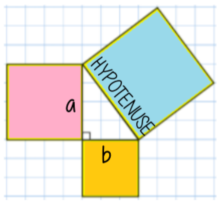
$Cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse side}}$

Substitute the values into the ratio formula

Equations might need rearranging to solve

Pythagoras theorem R

$Hypotenuse^2 = a^2 + b^2$



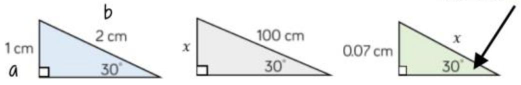
This is commutative – the square of the hypotenuse is equal to the sum of the squares of the two shorter sides

Places to look out for Pythagoras

- Perpendicular heights in isosceles triangles
- Diagonals on right angled shapes
- Distance between coordinates
- Any length made from a right angles

Ratio in right-angled triangles

When the angle is the same the ratio of sides a and b will also remain the same



a : b
1 : 2

$\frac{a}{b} = \frac{1}{2}$

a : b
x : 100
50 : 100

a : b
0.07 : x
0.07 : 0.14

Area of a triangle

Used when you know 2 sides and the angle between.

Formula: Area of triangle = ½ ab sinC

Note: This means ½ × b × c × sinA

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