Southmoor Academy
ASPIRE · ACHIEVE · ENJOY

The quadratic formula

For a general quadratic equation written $ax^2 + bx + c = 0$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

e.g. Solve $4x^2 - 8x - 7 = 0$

$a = 4 \quad b = -8 \quad c = -7$

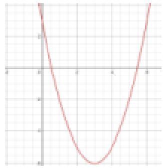
Be careful putting negatives into your calculator. Brackets around the negative number will help.

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \times 4 \times (-7)}}{2 \times 4}$$

$x = 2.66$ and $x = -0.66$ (2.d.p.)

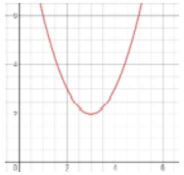
The $b^2 - 4ac$ is known as the **discriminant**.

If $b^2 - 4ac > 0$ then there are 2 unique roots. The graph crosses the x-axis in 2 places.



If $b^2 - 4ac = 0$ then there is a repeated root. The graph touches the x-axis in one spot.

If $b^2 - 4ac < 0$ there are no roots. The graph does not touch the x-axis.



Completing the square

Writing a quadratic equation in the form $(x + p)^2 + r = 0$ is known as completing the square.

e.g. Solve $x^2 + 6x - 8 = 0$

(Half the b value, so $6 \div 2 = 3$)

$(x + 3)^2 - 9 - 8 = 0$

(subtract this value squared as $(x + 3)^2$ multiplied out is $x^2 + 6x + 9$, not $x^2 + 6x$)

$(x + 3)^2 - 17 = 0$ (this is the completed the square form)

$(x + 3)^2 = 17$

$x + 3 = \pm\sqrt{17}$

$x = -3 \pm \sqrt{17}$

When written in the form $(x + p)^2 + r = 0$, you can determine key features of the graph.

The co-ordinate of the **turning point** of the curve (minimum/maximum point) is $(-p, r)$

Solving by factorising

Step 1: Rearrange the equation so that one side is equal to 0

Step 2: Factorise the equation

Step 3: Solve each factor equal to 0.

Solve $x^2 - 6x + 10 = 2$

$x^2 - 6x + 8 = 0$

$(x - 4)(x - 2) = 0$

Either $x - 4 = 0$ or $x - 2 = 0$

$x = 4$ and $x = 2$

e.g. Solve $2x^2 - 5x - 3 = 0$

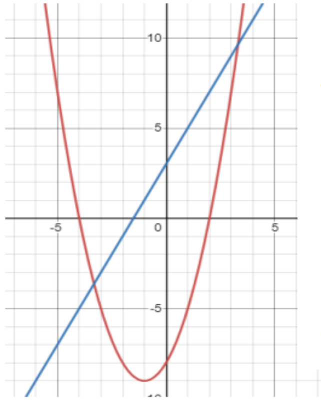
$(2x + 1)(x - 3) = 0$

Either $2x + 1 = 0$ or $x - 3 = 0$

$2x = -1$

$x = -\frac{1}{2}$ and $x = 3$

Simultaneous equations where one is non-linear



As a non-linear graph will curve, the solution to simultaneous equations with a non-linear equation can have more than 1 answer.

If we are solving a quadratic and a linear graph there are either:

- 0 solutions – the graphs do not intersect
- 1 solution – the linear graph is a tangent to the curve and touches only once
- 2 solutions – the graph crosses twice (as shown on the left)

Solving the equations algebraically allows us to find the exact values of these intersections.

e.g. Solve $y = x^2 + 3x - 8$

$y = 2x + 3$

As both equations are y =, we can equate them

$$x^2 + 3x - 8 = 2x + 3$$

Rearrange so that one side = 0

$$x^2 + x - 11 = 0$$

This does not factorise so using the formula

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times (-11)}}{2 \times 1}$$

x = 2.8541.. and x = -3.854...

Substitute these back into y = 2x + 3

y = 8.7082... and y = -4.708...

x = 2.86 and y = 8.71

x = -3.85 and y = -4.71

e.g. Solve $x^2 + y^2 = 10$ ← This is the equation of a circle

$y = 2x - 5$

This time we need to substitute y = 2x – 5 into the top equation.

$$x^2 + (2x - 5)^2 = 10$$

Multiply out the bracket

$$x^2 + 4x^2 - 20x + 25 = 10$$

Simplify and set one side = 0

$$5x^2 - 20x + 15 = 0$$

Factorise and solve

$$5(x^2 - 4x + 3) = 0$$

$$5(x - 3)(x - 1) = 0$$

x = 3 and x = 1

Substitute back into y = 2x – 5

when x = 3, y = 1

when x = 1, y = -3

Plotting and using quadratic graphs

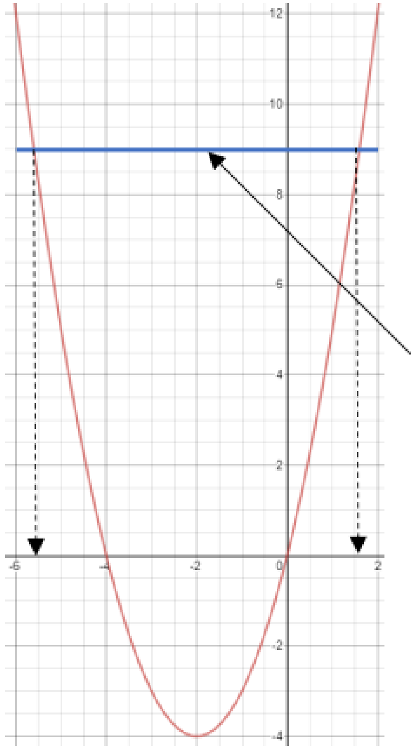
e.g. a) Complete the table of values for $y = x^2 + 4x$ and plot the graph

x	-6	-4	-2	0	2
y	12	0	-4	0	12

$y = (-6)^2 + 4 \times -6$

$y = 36 - 24 = 12$

As a quadratic graph is symmetrical, you will often see repeating values of y



b) Use the graph to find estimates for the solutions of $x^2 + 4x = 9$

We already have the graph of $y = x^2 + 4x$

We draw on to the same axis the graph of $y = 9$

Where the 2 graphs intersect (cross) we read off the two x values.

So x = 1.5 and x = -5.5

Southmoor Academy
ASPIRE · ACHIEVE · ENJOY