2E – Manipulating algebra – Stage 9

Algebraic Proof

Useful generalisations

Consecutive numbers	$n, n+1, n+2 \ldots$
Even numbers	$2n$
Odd numbers	$2n+1$
Consecutive evens	$2n, 2n+2, 2n+4 \ldots$
Consecutive odd	$2n+1, 2n+3, 2n+5 \ldots$

Prove the sum of any three consecutive numbers is a multiple of 3

Before you begin highlight the key words and information in the question

Prove the sum of any three consecutive numbers is a multiple of 3

Let the consecutive integers be n, $n+1$ and $n+2$

So $n + n + 1 + n + 2 = 3n + 3$

A multiple of three as 3 is a factor – $3(n+1)$

Identify equations, expressions, formulae and identities

Collection of terms with no equals sign

More than one variable and an equals sign

	Expression	Equation	Formula	Identity
$3x+4$	✓			
$3x+4=12$		✓		
$P=4x$			✓	
$3x+12 \equiv 3(x+4)$				✓

Has an equals sign and only one unknown. Can be solved.

Expanding a linear bracket

Multiply all terms inside the bracket by the term in front of the bracket being careful with any negative numbers

e.g. $4(3a-6) = 12a - 24$

as $4 \times 3a = 12a$ and $4 \times -6 = -24$

Algebraic fractions

Simplify the following:

$$\frac{x^2+7x-18}{4x-8}$$

Factorise first

$$\frac{(x+9)(x-2)}{4(x-2)}$$

Then cancel down

$$\frac{(x+9)(x-2)}{4(x-2)} = \frac{(x-2)}{4}$$

Simplify the following:

$$\frac{7}{x+3} + \frac{8}{x-4}$$

Multiply the fractions by each denominator

$$\frac{7(x-4)+8(x+3)}{(x+3)(x-4)}$$

Expand and simplify

$$\frac{7x-28+8x+24}{x^2+3x-4x-12} = \frac{15x-4}{x^2-x-12}$$

Factorise and cancel down again if possible

Simplify the following:

$$\frac{3(c+2)}{8c} \times \frac{2c(c-1)}{c+2}$$

Cancel down any common factors

$$\frac{3(\cancel{c+2})}{\cancel{8c}_{4}} \times \frac{\cancel{2c}(c-1)}{\cancel{c+2}}$$

Multiply the numerators then the denominators, cancel down if possible

$$\frac{3}{4} \times \frac{c-1}{1} = \frac{3(c-1)}{4} = \frac{3c-3}{4}$$

Simplify the following:

$$\frac{x+3}{3} + \frac{x-5}{2}$$

Find the LCM and multiply the fractions

$$\frac{2(x+3)+3(x-2)}{3 \times 2} = \frac{2x+6+3x-15}{6}$$

Then cancel down

$$\frac{5x+9}{6}$$

Simplify the following:

$$\frac{6x}{2y} \div \frac{4y}{5}$$

Find the reciprocal of the second fraction and multiply them

$$\frac{6x}{2y} \times \frac{5}{4y}$$

Take out any factors and cancel down

$$\frac{6x}{2y} \div \frac{4y}{5} = \frac{\cancel{6x}^{3}}{2y} \times \frac{5}{\cancel{4y}_{2}} = \frac{15}{4y^2}$$

Expanding a double bracket

Method 1

Draw loops between each pair and multiply the two values at the end of the loops together

$(2x+4)(3x+5)$

$2x \times 3x = 6x^2$

$4 \times 3x = 12x$

$2x \times 5 = 10x$

$12x + 10x = 22x$

$4 \times 5 = 20$

So $6x^2 + 22x + 20$

Method 2 – Separate the brackets

In this method we split the pair of brackets back into single ones

$(2x+4)(3x+5)$

$= 2x(3x+5) + 4(3x+5)$

$= 6x^2 + 10x + 12x + 20$

$= 6x^2 + 22x + 20$

Method 3 - Grid

Set the expansion out as a multiplication grid

$(2x+4)(3x+5)$

	$3x$	$+5$
$2x$	$6x^2$	$10x$
$+4$	$12x$	20

So $6x^2 + 22x + 20$

Rearranging Formulae (two step)

In an equation (find x)

$4x - 3 = 9$
$+3 \quad +3$
$4x = 12$
$\div 4 \quad \div 4$
$x = 3$

In a formula (make x the subject)

$xy - s = a$
$+s \quad +s$
$xy = a + s$
$\div y \quad \div y$
$x = \frac{a+s}{y}$

The steps are the same for solving and rearranging

Rearranging is often needed when using $y = mx + c$

eg Find the gradient of the line $2y - 4x = 9$

Make y the subject first $y = \frac{4x + 9}{2}$ Gradient $= \frac{4}{2} = 2$

Changing the subject of a formula where the wanted subject appears more than once

e.g. make b the subject of the formula

$a = \frac{2 - 7b}{b - 5}$ — $\times (b - 5)$

$a(b - 5) = 2 - 7b$ — expand the bracket

$ab - 5a = 2 - 7b$ — $+ 7b$

$ab + 7b - 5a = 2$ — $+ 5a$

$ab + 7b = 2 + 5a$ — factorise

$b(a + 7) = 2 + 5a$ — $\div (a + 7)$

$b = \frac{2 + 5a}{a + 7}$

Factorise into a single bracket $8x + 4$

$8x + 4$	4
$2x + 1$	

4 ← Try and make this the **highest common factor**

The two values **multiply** together (also the area) of the rectangle

$8x + 4 \equiv 4(2x + 1)$

Note:
$8x + 4 \equiv 2(4x + 2)$
This is factorised but the HCF has not been used

Factorising Quadratics

The general form of a quadratic expression is $ax^2 + bx + c$ where a, b and c are numbers.

Type 1: $a = 1$

When factorising a full quadratic expression, it goes into 2 brackets. The second terms in the brackets need to multiply to make the "+c" and add to make the "+b"

e.g.

$x^2 + 8x + 12$	$x^2 - 10x + 24$	$x^2 - 3x + 28$
$6 \times 2 = 12$ and $6 + 2 = 8$	$-6 \times -4 = 24$ and $-6 + -4 = -10$	$-7 \times 4 = -28$ and $-7 + 4 = -3$
$(x + 6)(x + 2)$	$(x - 6)(x - 4)$	$(x - 7)(x + 4)$

Special cases: 1) No "+c" e.g. $6x^2 + 3x$ This factorises into 1 bracket rather than 2. $6x^2 + 3x = 3x(2x + 1)$

2) No "+b" and c is negative e.g. $x^2 - 25$ This is known as the **difference of two squares** and factorises into two brackets. Both brackets are the same except the sign in the middle $x^2 - 25 = (x + 5)(x - 5)$

Type 2: $a > 1$

e.g. $6x^2 - 11x - 10$

Step 1 – multiply a and c together then find factors of this number which add to b

$6 \times -10 = -60$. Factors of -60 which add to -11 are -15 and +4

Step 2 – Rewrite the b term (-11x) using these two factors

$6x^2 - 15x + 4x - 10$

Step 3 – Factorise the first two terms into one bracket

Step 4 – Factorise the last two terms into one bracket. Tip – it will be the same bracket as used for the first two terms

$3x(2x - 5) + 4x - 10$

$3x(2x - 5) + 2(2x - 5)$

Step 5 – This bracket is a factor of both terms so now rewrite as two brackets

$(3x + 2)(2x - 5)$

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