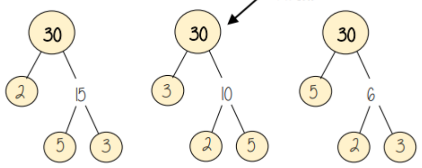
Product of prime factors

Multiplication part-whole models



All three prime factor trees represent the same decomposition

Multiplication is commutative

30 = 2 x 3 x 5 ← Multiplication of prime factors

Using prime factors for predictions

eg 60	30 x 2	2 x 3 x 5 x 2
150	30 x 5	2 x 3 x 5 x 5

Finding the HCF and LCM

HCF – Highest common factor

HCF of 18 and 30

18: 1, 2, 3, 6, 9, 18

30: 1, 2, 3, 5, 6, 10, 15, 30

6 is the biggest factor they share

HCF = 6

LCM – Lowest common multiple

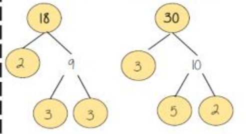
LCM of 18 and 30

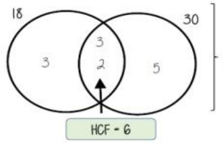
18: 18, 36, 54, 72, 90

30: 30, 60, 90

The first time their multiples match

LCM = 90





LCM = 90

Addition/ Subtraction laws for indices

$3^5 \times 3^2$ → 3^7

= (3 x 3 x 3 x 3 x 3) x (3 x 3)

The base number is all the same so the terms can be simplified

Addition law for indices

$a^m \times a^n = a^{m+n}$

$3^5 \div 3^2$ → 3^3

$\frac{3 \times 3 \times 3 \times 3 \times 3}{3 \times 3}$ → $\frac{3^3}{3^0}$ → $\frac{3^3}{1}$

Subtraction law for indices

$a^m \div a^n = a^{m-n}$

Order numbers in standard form

10^2	10^1	10^0	•	10^{-1}	10^{-2}	10^{-3}	10^{-4}

6.4×10^{-2}	2.4×10^2	3.3×10^0	1.3×10^{-1}
0.064	240	1	0.13

Look at the power first will the number be = > or < than 1

Use a place value grid to compare the numbers for ordering

Standard form with numbers > 1

Any number between 1 and less than 10 → $A \times 10^n$ ← Any integer

Example

3.2×10^4

= 3.2 x 10 x 10 x 10 x 10

= 32000

Non-example

0.8×10^4

$5.3 \times 10^{0.7}$

Numbers between 0 and 1

0.054

$= 5.4 \times 10^{-2}$

1	•	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^0	•	10^{-1}	10^{-2}	10^{-3}
0	•	0	5	4

A negative power does not mean a negative answer – it means a number closer to 0

Zero and negative indices

$$x^0 = 1$$

Any number divided by itself = 1

$\frac{a^6}{a^6} = a^6 \div a^6$

$= a^{6-6} = a^0 = 1$

Negative indices do not indicate negative solutions

$2^2 = 4$

$2^1 = 2$

$2^0 = 1$

$2^{-1} = \frac{1}{2}$

$2^{-2} = \frac{1}{4}$

Looking at the sequence can help to understand negative powers

Powers of powers

$$(x^a)^b = x^{ab}$$

$(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3$

The same base and power is repeated Use the addition law for indices

$(2^3)^4 = 2^{12}$ ← $a \times b = 3 \times 4 = 12$

NOTICE the difference

$(2x^3)^4 = 2x^3 \times 2x^3 \times 2x^3 \times 2x^3$

The addition law applies ONLY to the powers. The integers still need to be multiplied

$(2x^3)^4 = 16x^{12}$

Higher powers and roots

x^n

n – power (number of times multiplied by itself)

x – the base number.

$\sqrt[n]{x}$ ← Finding the nth root of any value

Other mental strategies for square roots

$\sqrt{810000} = \sqrt{81} \times \sqrt{10000}$

$= 9 \times 100$

$= 900$

Multiplication and division

$\frac{1.5 \times 10^5}{0.3 \times 10^3}$ ← Division questions can look like this

$(1.5 \times 10^5) \div (0.3 \times 10^3)$

$(1.5 \div 0.3) \times 10^5 \div 10^3$

$= 5 \times 10^2$

For multiplication and division you can look at the values for A and the powers of 10 as two separate calculations

Revisit addition and subtraction laws for indices – they are needed for the calculations

Addition law for indices

$a^m \times a^n = a^{m+n}$

Subtraction law for indices

$a^m \div a^n = a^{m-n}$

Addition and Subtraction

Tip: Convert into ordinary numbers first and back to standard from at the end

$6 \times 10^5 + 8 \times 10^5$

Method 1

= 600000 + 800000

= 1400000

$= 1.4 \times 10^5$

More robust method

Less room for misconceptions

Easier to do calculations with negative indices

Can use for different powers

Method 2

$= (6 + 8) \times 10^5$

$= 14 \times 10^5$

$= 1.4 \times 10^1 \times 10^5$

$= 1.4 \times 10^5$

This is not the final answer

Only works if the powers are the same

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